Abstract: This project evaluates the performance of improved power delay profile estimation in Multiple input multiple output orthogonal frequency division multiplexing (MIMO-OFDM) systems under linear minimum mean square error (LMMSE) channel. The distortions caused by null subcarriers and a short range of samples for PDP estimation is also considered. The proposed technique effectively estimates the distortions caused by PDP. Simulation results show that the performance of extended Kalman filtering estimation using the different PDP is more appropriate. The output is compared against different antenna systems and with different delay profiles along with the conventional LMMSE.

Keywords: channel estimation, power delay profile, MIMO, OFDM, 3GPP-LTE, Extended Kalman filtering

I. INTRODUCTION

Wireless communication systems in which Multiple-Input Multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) is one of the most promising techniques, including the third-generation partnership project long-term evolution (3GPP LTE) and IEEE 802.16 (WiMAX). MIMO-OFDM systems provide a better spatial diversity or multiplexing gain. Most receiver techniques of MIMO-OFDM systems are designed with the channel state information (CSI), so as to achieve the maximum diversity or multiplexing gain. The most of performance gain depends heavily on correct channel estimation that is most important for the MIMO-OFDM systems.

When the receiver knows the channel statistics such as the pilot-aided channel estimation, based on the linear minimum mean square error (LMMSE) technique. The power delay profile estimation schemes have been proposed to obtain the frequency domain channel statistics at the receiver, the Maximum likelihood (ML) estimation is a one of the PDP estimation technique for LMMSE channel by taking advantage of the cyclic prefix (CP) segment of OFDM symbols. But it requires very high computational complexity for obtaining an accurate PDP.

To improving the performance of LMMSE channel estimation employs another approach (i.e. uniform or exponential model) with the estimation of second channel Statistics they are mean delay and root-mean-Square (RMS) Delay spread. By using the pilots we can estimate the channel delay parameters with the low computational complexity. Therefore, the LMMSE channel estimator with the approximated PDP is suitable for sensible applications like a WiMAX system. However, the correlation mismatch and estimation error of delay parameters are caused by the degradation performance in approximated PDP estimation technique.

The signal represented in frequency domain we can minimize the signal mismatch. We can evaluate the improved power delay profile estimation in MIMO-OFDM systems under LMMSE channel. For sensible applications, alone the pilot symbols of all transmit antenna ports area units are used in estimating the PDP with low computational complexity. Additionally, the proposed technique effectively minimizes the distortion effects, incurred by null subcarriers and a short range of channel impulse response (CIR) samples for PDP estimation is also considered. The proposed technique effectively minimizes the distortions for proper PDP estimation. Simulation results show that the performance of LMMSE channel estimation using the different PDP estimation techniques. The output of LMMSE channel by using the proposed PDP estimate approaches that of Wiener filtering as results of the mitigation of distortion effects.

II. SYSTEM MODEL

Let us consider a MIMO-OFDM system with P Transmit and S receives antennas, and K total subcarriers. To control interference with other system the MIMO-OFDM system transmits $K_d$ subcarriers at the middle of spectrum located for both data and Pilots, with K-$K_d$ virtual subcarriers. In a MIMO systems having the same PDP of CIR corresponding to different transmit and receive antennas.

Let the pilot subcarrier for the $p^{th}$ transmit antenna at the $n_p^{th}$ OFDM symbol represented by $C_p [k_p n_p]$, that could be a QPSK modulated signal from known sequences between the transmitter and receiver. We assume that the
pilot subcarriers are distributed over a time and frequency grid as in Fig. 1, to define the Orthogonality of pilots among different transmits antennas. \( k_p \in f_p \) And \( n_p \in T_p \) represent the index sets for the

Figure 1: Pilot symbol arrangement in a physical resource block (PRB) of the LTE OFDM system.

III. PROPOSED METHOD FOR THE PDP ESTIMATION

A. Derivation of the PDP in MIMO-OFDM systems

From (1), the CIR at the \((p,S)\)th antenna port can be estimated approximately using the regularized least squares (RLS) channel estimation with a fixed length of \( L_g \) as

\[
\hat{h}_{R,p,s} = (G_p^H G_p + X_{RLS,p} \epsilon |L_B|^{-1})^{-1} \text{diag}(W_p) H_y n_p
\]

(2)

Where \( \epsilon = 0.001 \) is a small regularization parameter, and \( L_g \) is the \( L_g \times L_g \) identity matrix. \( G_p^H G_p \) in (2) is ill-conditioned due to the sparsely of pilot tones in the frequency domain and the presence of virtual subcarriers.

To derive the PDP from the estimated CIR in (2), the ensemble average of \( \hat{h}_{R,p,s} \hat{h}_{R,p,s}^H \) is given by

\[
E(\hat{h}_{R,p,s} \hat{h}_{R,p,s}^H) = XR_{hh} X^H + \sigma^2 X_{RLS,p} X_{RLS,p}^H
\]

(3)

Where \( R_{hh} = (h_{p,s} h_{p,s}^H) \) and \( X=(G_p^H G_p \otimes |L_B|)^{-1} G_p^H G_p \). Note that the diagonal elements of the channel covariance matrix, \( R_{hh} \), represent the PDP of multipath channel within the length of \( L_g \), and all off-diagonal elements are zeros. Hence, the covariance matrix can be expressed as \( R_{hh} = \text{diag} \left( R_{ss} \right) \), where \( R_{ss} = \left[ p_{11}, p_{12}, p_{13}, \ldots, 0 \right] \) and \( p_{1} = E \left[ \{ h_{p,s}[n_p,l]\}^\dagger \right] \). Unfortunately, it is \( R_{hh} \) distorted by \( X \), which is an ill-conditioned matrix due to the presence of \( p \). Thus, instead of calculating \( X^{-1} \), we investigate the method for eliminating the spectral leakage of \( X \). The covariance matrix of the estimated CIR is defined as \( R_{hh} = X R_{hh} X \) which can be expressed as

\[
R_{hh} = \sum_{l=0}^{L_g-1} \text{diag}(R_l u_l^H) X^H
\]

(4)

Where \( u_l \) is a unit vector with the \( l \)th entry being one and otherwise zeros. Let \( p_h \) and \( t_l \) be the \( L_g \times 1 \) vectors defined as \( p_h = (R_{hh}) \) and \( t_l = D_g \times \text{diag}(u_l) X^H \), respectively.
Where $D_p(X)$ column vector contain all the diagonal elements of a then the relation in (4) is simplified as

$$p_h = p_0 t_0 + p_1 t_1 + \ldots + p_{Lg-1} t_{Lg-1} = Tp_h$$ (5)

where $T=[t_0, t_1, t_2, \ldots, t_{Lg-1}]$ is defined as a distortion matrix by $X$. it is noted that the distortion matrix is a strictly diagonally dominant matrix, satisfying the non-diagonal elements of $T$ are composed of the leakage power of $u_j$ for all $j$ from the Gersgorin circle theorem, a strictly diagonally dominant matrix is non-singular. In addition, the distortion matrix is a well-conditioned matrix. Hence, the distortion of $w$ can be eliminated as

$$p_h = T^2 p_h = E\{g(p_h|n_p)\} - \sigma_n^2 \bar{X},$$ (6)

Where $\{g(p_h|n_p)\} = T^2 Dg(h_{p,s} h_{p,s}^H)$ is defined as the received sample vector for estimating PDP at the $(p,s)$Th antenna port on the $n_p$Th OFDM symbol, and $\bar{X} = T^2 Dg(X_{RLS,p} X_{RLS,p})$.

**B. PDP Estimation in Practical MIMO-OFDM Systems**

The received sample vector in (6) can be expressed as

$$[n_p] = T^{-1} Dg(h_{p,s} h_{p,s}^H) + \hat{n}_{p,s} + e_{p,s}$$ (7)

where $\hat{n}_{p,s} = T^{-1} Dg(X_{RLS,p} n_p X_{RLS,p})$ and $e_{p,s} = 2Re\{T^{-1} Dg(X h_{p,s} n_p X h_{p,s}^H)\}$, here, $Re\{a\}$ denotes the real part of $a$. We assume that $\hat{n}_{p,s}$ is an effective noise by AWGN.then the sample average of $g(p,s|n_p)$ is given

$$(g_p|n_p)_N \triangleq \frac{1}{N} \sum_{n=1}^{N} \sum_{p=1}^{P} \sum_{s=1}^{S} g(p,s|n_p)$$ (8)

Where $\hat{n}_{p,s} = T^{-1} Dg(h_{p,s} h_{p,s}^H) + \hat{n}_{p,s} + e_{p,s}$

$$[n_p] = T^{-1} Dg(h_{p,s} h_{p,s}^H)|N \rightarrow p_h, \hat{n}_{p,s}|N \rightarrow \sigma_n^2 \bar{X}, \text{ and } e_{p,s}|N \rightarrow 0.$$

However, it is difficult for a receiver of practical MIMO-OFDM systems to obtain such a large number of samples. With an insufficient number of samples, the PDP can be approximated as $p_h \approx (Dg(h_{p,s} h_{p,s}^H))|N$. To improve the accuracy of PDP estimation with insufficient samples, we mitigate the effective noise as follows

$$(g_p|n_p)_N - \sigma_n^2 \bar{X}, = (Dg(h_{p,s} h_{p,s}^H))|N + z_n$$ (9)

Where $z_n \triangleq (\hat{n}_{p,s})|N + (e_{p,s})|N - \sigma_n^2 \bar{X}$, is defined as a residual noise vector, in which each entry has a zero-mean. Then, the error of PDP estimation with $N$ samples can be calculated as

$$\hat{e}_N = (Dg(h_{p,s} h_{p,s}^H))|N - p_h + z_N$$ (10)

Where $[p_h]_j \geq 0$ for all $j$, the PDP can be initially estimated as

$$\hat{p}_{init} = \frac{1}{N} \sum_{n=1}^{N} [g_p|n_p]$$ (11)

Where $Q_{p,s}|n_p$ is the sample vector of proposed PDP with the $l_s$ entry

$$Q_{p,s}|n_p = \{[g_p|n_p]|n_p - \sigma_n^2 \bar{X}\} \text{ if } g_{p,s}|n_p > \sigma_n^2 \bar{X}\text{ otherwise}$$ (12)

where $g_{p,s}|n_p = [g_p|n_p]|1$ and $\bar{X} = [\bar{X}]_1$ to mitigate the detrimental effect of residual noise $Z_N$, the proposed scheme estimates the average of residual noise at the zero-taps of $p_h$. At the $l_s$th entry of $z$, the zero-tap can be detected as $p_{init}$, the zero-tap can be detected as

$$t_z^l = \begin{cases} 1 & \text{if } p_{init} < \beta_{th} \\ 0 & \text{otherwise} \end{cases}$$ (13)

Where $\beta_{th} = \frac{1}{Lg} \sum_{l=0}^{\text{lg-1}} p_{init}$ is defined as a threshold value for the zero-tap detection. Then, the average of residual noise at the zero-taps can be estimated as

$$\hat{n}_{R,avg} = \frac{1}{Nz} \sum_{l=0}^{\text{lg-1}} p_{init} t_z^l$$ (14)

Where $N_z = \sum_{l=0}^{\text{lg-1}} t_z^l$ represents the total number of detected zero-taps. With the mitigation of residual noise, the $A$th tap of the PDP estimate, $\hat{p}_h$ can be expressed as

$$\hat{p}_h = \begin{cases} p_{init} - \hat{n}_{R,avg} & \text{if } p_{init} > \hat{n}_{R,avg} \\ 0 & \text{otherwise} \end{cases}$$ (15)

Then, the estimated PDP in (15) can be used to obtain the frequency-domain channel correlation in the LMMSE channel estimator.

**IV. EXTENDED KALMAN FILTERING**

Let us consider a MIMO OFDM system with $N_t$ antennas the relation between the transmit and received signal can be expressed as

$$X[n,k] = P[n,k] H[n,k] + n[n,k]$$ (16)

Where $P[n,k]$ denotes the Pilot symbols then
Thus the channel state information can be achieved by applying the inverse Fast Fourier transform to the transfer function and we can conclude that

\[ h_t[n] = h_c[n] + Z_t[n] \] (18)

Where \( Z \) is a zero mean complex Gaussian vector distribution is \( \mathcal{N}(0, \sigma^2) \) [11][12]

V. SIMULATION RESULTS

We consider a MIMO-OFDM system with the physical layer parameters for the downlink of 3GPP LTE. The MIMO-OFDM system utilizes four transmit and two receive antennas (\( P=4, S=2 \)). We assume that the pilots of the transmit antenna ports are distributed as the time and frequency grid of the LTE system physical resource block shown in Fig.1. The system bandwidth is 5 MHz with 301 subcarriers for transmitting both data and pilots including DC subcarrier at 2 GHz carrier frequency. The width of each subcarrier is 15 kHz with an FFT size of 512. The length of CP is forty (\( L_g=40 \)). For all simulations, the channel estimator is based on cascaded 2x1D LMMSE technique throughout 14 OFDM symbols (\( |T1| = |T2| = 2 \), \( |T3| = |T4| = 1 \)), as shown in Fig.1 wherever the filtering in frequency domain is followed by the filtering in time domain over slowly fading channels with the Doppler frequency of 5 Hz.

Figure 2 shows the performance of LMMSE technique using the estimated PDP and Kalman filtering over extended typical urban channel with variable channel lengths.

The performance of 2x1D LMMSE technique using the approximated PDP which is uniform or exponential model with the channel delay parameter estimation is also plotted. The performance of the 2x1D wiener filter with exact PDP is included as a lower bound. For performance comparisons we plot the performance of frequency domain regularized LS Channel estimation in which the PDP information is not required. Note that by using the proposed PDP estimation Technique we can reduce the correlation mismatch in MIMO-OFDM systems. In Figure 3 we investigate the performance of the proposed theme over the exponentially power decaying six path Rayleigh fading channel model wherever the channel maximum delay, \( L_{ch} \) is Variable. The PDP of the channel model is defined as \( E[\alpha_{\text{PDP}}[n]]^2 = \frac{1}{\text{s}_h}e^{tr_{\text{rms}}} \) for \( t=0, \Delta t, \ldots, 5\Delta t \) and \( \Delta t = \frac{L_{ch}}{5} \). Here \( t_{\text{rms}} = \frac{L_{ch}}{\log(2L_{ch})} \) and \( s_h \) is the normalization.
factor \((r = \sum_{i=1} e^{j\phi_{mi}})\). The performance of the proposed scheme is better than that of the conventional methods, and approaches that of Wiener filtering in various channel environments. Figure 4 shows it can be seen that the MSE of LMMSE technique using the estimated PDP achieves that of Wiener filtering even at high Doppler frequencies. The MSE performance of the \(2x1D\) LMMSE technique using the estimated PDP for different mobile Equipment speeds at 30-dB SNR. All underlying links are modeled as ETU channels. We assume that to \(2x2\) MIMO-OFDM system over ETU channels with 70-Hz. doppler frequency. The simulation results correspond to the channel estimation performance at the first OFDM symbol of antenna port 1 shown in Fig. 1. We obtain to get the analytic results in (22) by using the coefficient matrix for LMMSE channel estimation with the perfect or imperfect PDP at the antenna port. In Fig. 5, it’s determined that the MSE of the proposed theme improves the MSE performance with an increase in the number of samples for PDP estimation.

VI. CONCLUSION

We proposed an improved power delay profile estimation in multiple inputs multiple output orthogonal frequency division multiplexing (MIMO-OFDM) systems under linear minimum mean square error (LMMSE) channel. The CIR estimates at every path of the MIMO channels were used to obtain the PDP. For correct PDP estimation, we consider the spectral leakage effect from virtual subcarriers, and also the residual noise caused by the insufficient number of estimated CIR samples. The proposed technique effectively reduces the both spectral leakage and residual noise. Simulation results show that the performance of LMMSE channel estimation using different PDP estimation techniques .the outputs of proposed PDP estimate approaches that of extended Kalman filtering.

REFERENCES

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